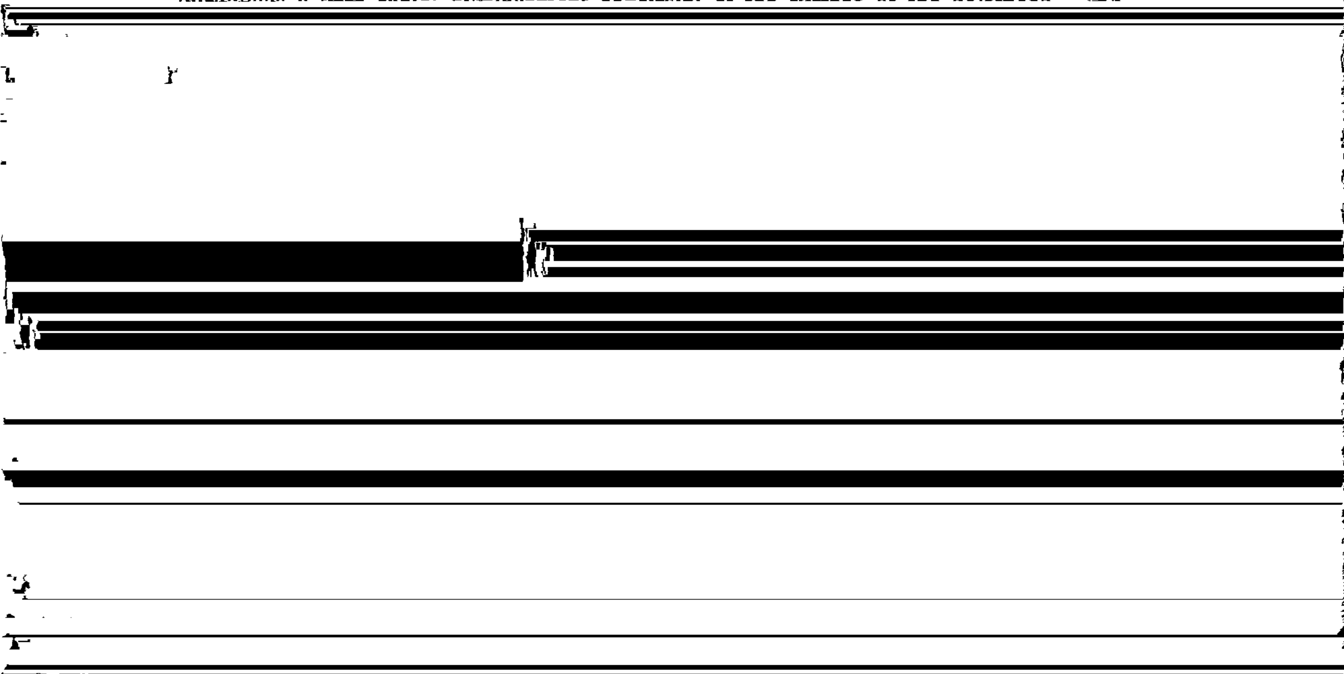
2 Axisymmetric problem for a surface constrained halfspace

We consider the axisymmetric problem of a thin plate of thickness t and infinite extent which is bonded to the surface of a halfspace and loaded by an external axisymmetric load $p(r)$ and a Mindlin force of magnitude P_M, which is located at a distance h from the bonded plate (Figure 1). The plate is assumed to be *inextensible* in its plane; this introduces a zero radial displacement constraint at the surface of the halfspace. The



objective of the preliminary analysis is to develop the relationship between an applied axisymmetric surface normal stress and the corresponding axisymmetric surface displacement in the axial direction. The solution of this class of problem can be approached by appeal to Love's [21] strain function ($\Phi(r, z)$)(See also Selvadurai [22]) formulation, where the governing partial differential equation is

$$\nabla^2\nabla^2\Phi(r,z)=0 \tag{1}$$

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \tag{2}$$

is Laplace's operator in axisymmetric cylindrical polar coordinates. By adopting a

[redacted]

$$D\xi^4 \overline{w}^0(\xi) + \overline{q}_c^0(\xi) = \overline{p}^0(\xi) \tag{13}$$

The relationship between the surface displacement of the halfspace due to the combined

For example, in order to evaluate the displacement $w(0)$ of the stiffening surface plate we need to evaluate integrals of the type

$$I = \int_0^{\infty} \frac{\xi\, e^{-\lambda\xi}}{\left[1+\xi^3\right]} d\xi \tag{20}$$

Although symbolic manipulations through the use of software such as MAPLE™ and MATHEMATICA™ can be used to represent the integral (20) in a compact form as

$$I = -\frac{1}{3} e^{\lambda} Ei(1,\lambda) + \frac{e^{-\lambda/2}}{6\sqrt{C_1}} \left\{ C_1 C_2\, Ei(1,\overline{C}_3) + \overline{C}_2\, Ei(1, C_3) \right\} \tag{21}$$

where

$$\begin{aligned} C_1 &= (-1)^{\lambda\sqrt{3}/\pi}; \\ C_2 &= 1 + i\sqrt{3}; \\ C_3 &= -\frac{\lambda}{2} C_2 \end{aligned} \tag{22}$$

$\overline{C}_2$ and $\overline{C}_3$ are complex conjugates and $Ei(n,x)$ is the exponential integral defined by