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where $A_i(\zeta)$ are unknown functions and $\lambda_i = \zeta/a_{\zeta}/v_i$. From (5) and (12) we note that in order to satisfy the boundary condition (8) we require

$$
\sqrt{v_2} A_1(\xi)(1 + k_1) = -\sqrt{v_1} A_2(\xi)(1 + k_2)
$$
\n(13)

Making use of this result and Eqns. (3), (4) and (12) it can be shown that the boundary conditions (9) and (10) are equivalent to the system of dual integral equations:

$$
\int_0^\infty \zeta B(\zeta) F(\zeta) J_0(\zeta r/a) d\zeta = \frac{p(r)}{2\mu^*}; \quad 0 \le r \le a \tag{14}
$$

$$
\int_0^\infty B(\xi) J_0(\xi r/a) d\xi = 0; \quad a < r < \infty \tag{15}
$$

where

$$
B(\xi) = \xi^2 A_2(\xi); \quad F(\xi) = 1 - \frac{\psi}{\xi}
$$

$$
\int_{-\frac{\xi - \mu^2 h}{\xi}}^{\frac{\xi - \mu^2 h}{\xi}} \frac{\xi \sqrt{\mu^2 h^2 - 1}}{\xi \sqrt{\mu^2 h^2 - 1}}
$$

5. The stress intensity factor

A result of primary interest to linear elastic fracture mechanics of the fibre reinforced composite concerns the distribution of stress in the vicinity of the boundary of the bridged flaw region. This state of stress is characterized by the stress intensity factor K_1 (for the flaw opening mode) defined by

$$
K_1 = \lim_{r \to a^+} \left[2(r - a) \right]^{\frac{1}{2}} \sigma_{zz}(r, 0) \tag{20}
$$

By employing the results derived in the previous section it can be shown that

$$
[K_{\text{1}}]_{\text{bridgedflaw}} = \frac{P}{\pi^2 a^{3/2}} \left\{ 2\Phi^*(1) \right\} \tag{21}
$$

In the limiting case when the elasticity of the bridging fibres E_f reduces to zero, $\psi = 0$, and (18) gives the result

$$
[K_{1}]_{\text{unbridgedflaw}} = \frac{P}{\pi^{2}a^{3/2}} \frac{(c_{13} + c_{44})v_{1}v_{2}}{c_{33}c_{44}(v_{1} - v_{2})} \times \frac{\left[(k_{1}c_{33} - v_{1}c_{13}) - (k_{2}c_{33} - v_{2}c_{13})\right]}{\left[\frac{(k_{1}c_{33} - v_{1}c_{13})}{\cdots} - \frac{(k_{2}c_{33} - v_{2}c_{13})}{\cdots}\right]}
$$
(22)

 \sim 23 \sim 24 \sim

where 2 and 2 and 2 are the classical Lam6 constants. Thus for the isotropic case of the body force of the body force

In the limit material isotropy v_1 , $v_2 \rightarrow 1$ and

 $\mathcal{F}^{\mathcal{F}}_{\mathcal{F}}$ (1, $\mathcal{F}^{\mathcal{F}}_{\mathcal{F}}$) and $\mathcal{F}^{\mathcal{F}}_{\mathcal{F}}$ (1, $\mathcal{F}^{\mathcal{F}}_{\mathcal{F}}$), where $\mathcal{F}^{\mathcal{F}}_{\mathcal{F}}$

In the limit material isotropy vl, v2 ~ 1 and

$$
c_{11} = c_{33} = \lambda + 2\mu; \quad c_{13} = \lambda; \quad c_{44} = \mu \tag{23}
$$

where λ and μ are the classical Lamé constants. Thus for the isotropic case of the body force loading of a penny shaped flaw

$$
[K_1]_{\text{unbridge-dflaw}}^{\text{isotropic}} = \frac{P}{\pi^2 a^{3/2}} \left[\frac{(1 - v) + \eta^2 (2 - v)}{(1 - v)(1 + \eta^2)^2} \right]
$$
(24)

This is in agreement with the result given by Kassir and Sih [22] and Barenblatt [31]. Also as

or stress channelling at the flaw boundary cannot be excluded (see e.g. Morland β

 $\eta \rightarrow 0$, the results (22) and (24) both yield the same result

This integral equation has a trivial solution $\mathcal{C}(\mathcal{C})$, $\mathcal{C}(\mathcal{C})$ and $\mathcal{C}(\mathcal{C})$ and $\mathcal{C}(\mathcal{C})$

the factor ~ ~ ~ and the integral equation (18) reduces to

$$
\Gamma K_{\nu} \Gamma_{\nu}^{\text{isotropic}} = \Gamma K \Gamma_{\text{transversely isotropic}} = \frac{P}{\Gamma K}
$$
 (25)

[33] and Pipkin [34]).

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