


be subjected to either concentrated or distributed force systems which act on the boundary or at the interior of the orthotropic quarter-plane. The elastic quarter-plane constitutes a special case of the more general class of elastic wedge problems which have received considerable attention. The two dimensional problems of



surface. Craft and Richardson [26] have also applied Hetenyi's method to obtain the state of stress in an isotropic quarter-plane containing a circular inclusion. The superposition procedure has also been extended by Hetenyi [27] to obtain solutions to the elastic quarter-space subjected to concentrated forces.

Hetenyi's method for the solution of the quarter-plane problem is not general in character. In the case of a concentrated force

problem one has to start with the solution to the appropriate half-plane problem and superpose a series of infinite integrals at each stage to satisfy the traction boundary conditions. Nevertheless the

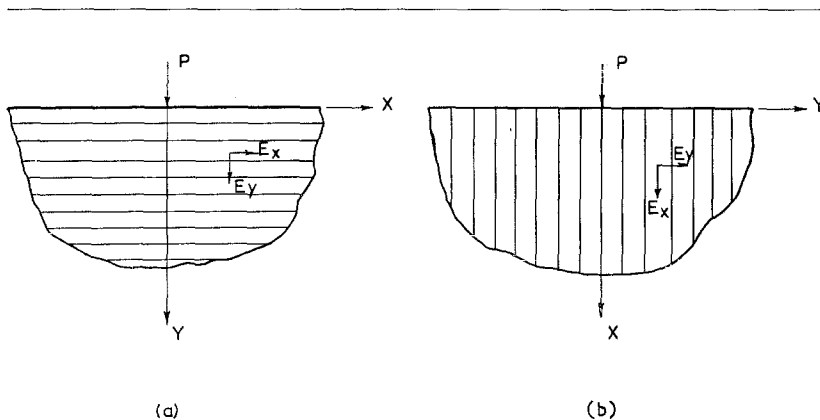


Fig. 1. Orthotropic elastic half-plane subjected to a concentrated force.

nents derived from (10) and (5) can be written in the form

$$[\sigma_{xx}, \sigma_{yy}, \sigma_{xy}] = \frac{P(k_1 + k_2)}{[x^2 + y^2]^{3/2}} [x^2 - y^2, x^2 + y^2, -2xy] \quad (11)$$

where

$$x = X/a, \quad y = Y/a,$$

are the non-dimensional spatial coordinates and a is a typical length parameter. Similarly, in the particular case of an orthotropic half-plane occupying the region $X > 0$ (Fig. 1b) the state of stress due to a concentrated force acting at the origin in the X -direction is given by the stress function

are zero on the plane of symmetry. The plane $X=0$ is therefore

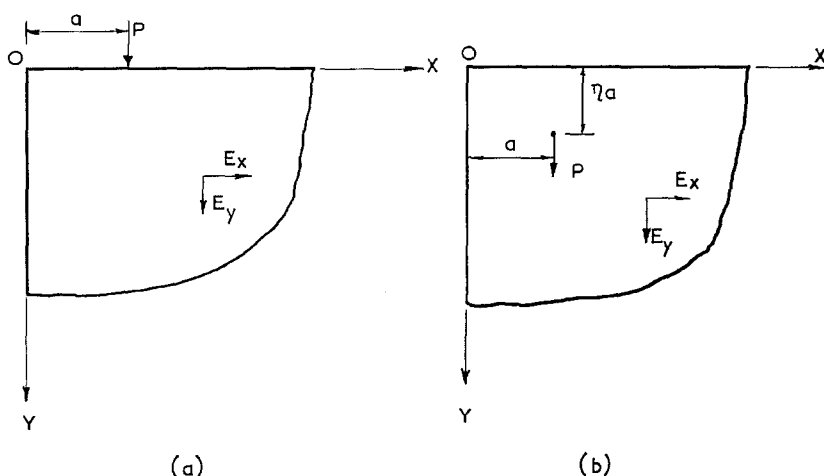
subjected to only a purely normal stress, $F_0(\bar{y})$. If an additional state of stress can be found such that in relation to the quarter-plane occupying the first quadrant (Fig. 1a), the shear tractions on planes $X=0$ and $Y=0$ are identically zero and the normal tractions on planes $X=0$ and $Y=0$ are $-F_0(\bar{y})$ and zero respectively, then this additional state of stress together with the basic

where

and

$$[J_{xx}^{\pm}(\bar{y}); J_{yy}^{\pm}(\bar{y}); J_{xy}^{\pm}(\bar{y})] = \frac{[x^3; x(y \pm \bar{y})^2; x^2(y \pm \bar{y})]}{[k_1^2 x^2 + (y \pm \bar{y})^2][k_2^2 x^2 + (y \pm \bar{y})^2]}. \quad (17)$$

Thus, combining the stress components derived from Step I with those of the basic state of stress renders the plane $X = 0$ free of normal traction but gives rise to a non-zero normal traction $F_1(\bar{x})$ on the plane $Y = 0$. To eliminate $F_1(\bar{x})$ we consider the symmetric state of external normal stress $-F_1(\bar{x})$ on the plane $Y = 0$ for the half-plane $Y > 0$ (Step 2). Again, the complete stress components



a) Concentrated force acting normal to the boundary

Consider the problem of a concentrated force, P , applied on the boundary of the orthotropic quarter-plane at a distance a from the origin (Fig. 3a). The basic state of stress can be obtained by combining the results for two concentrated normal forces, acting equidistant from the origin, on the surface of the half-plane. The basic state of stress can be written as

$$\sigma_{ij}^{(0)} = \frac{P(k_1 + k_2)}{\pi a} S_{ij}, \quad (26)$$

where

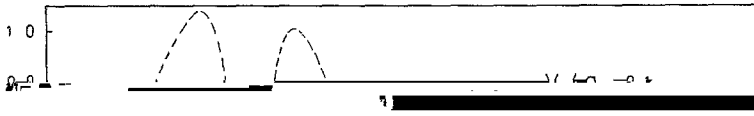
$$S_{ij} = S_{ij}^+ + S_{ij}^-.$$

and

$$\Gamma(\alpha, \beta) = \frac{1}{2\pi} \int_0^{2\pi} (\alpha + \beta \cos \theta)^{-1} d\theta.$$

logarithmic scale using Simpson's rule. The reversal procedure of these boundary stresses were carried out up to forty cycles which

provided in the resulting boundary stress components an accuracy of at least ten correct decimals. The corrective state of stress $\sigma_{ij}^{(e)}$ at a point in the quarter-plane is calculated by combining the stresses induced in the respective half-planes due to these boundary



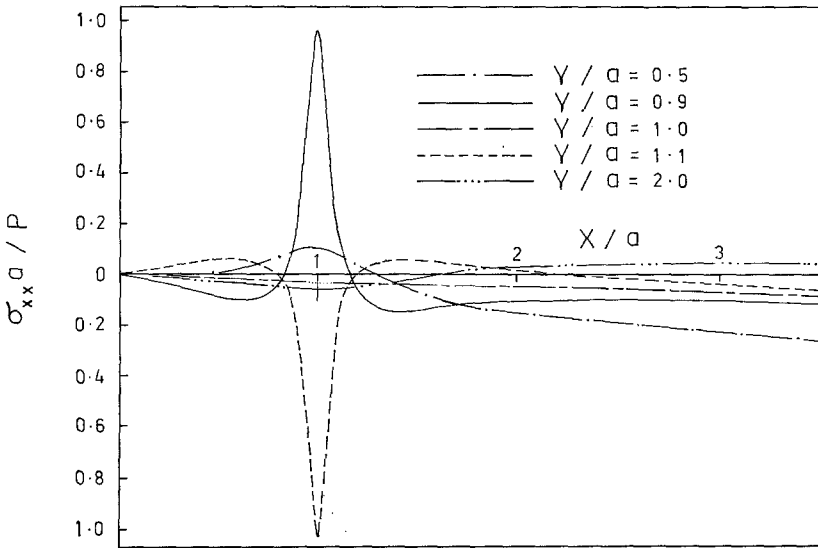
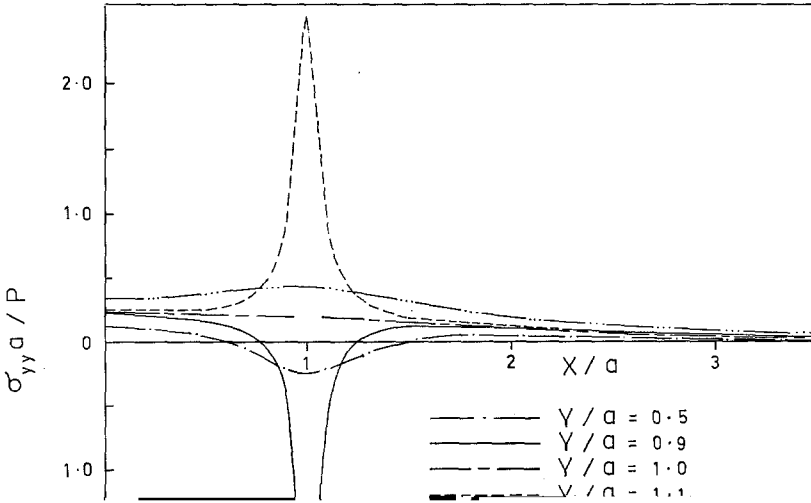


Fig. 10. Interior force - Variation of σ_{xx} with X/a - Graphite-epoxy composite.



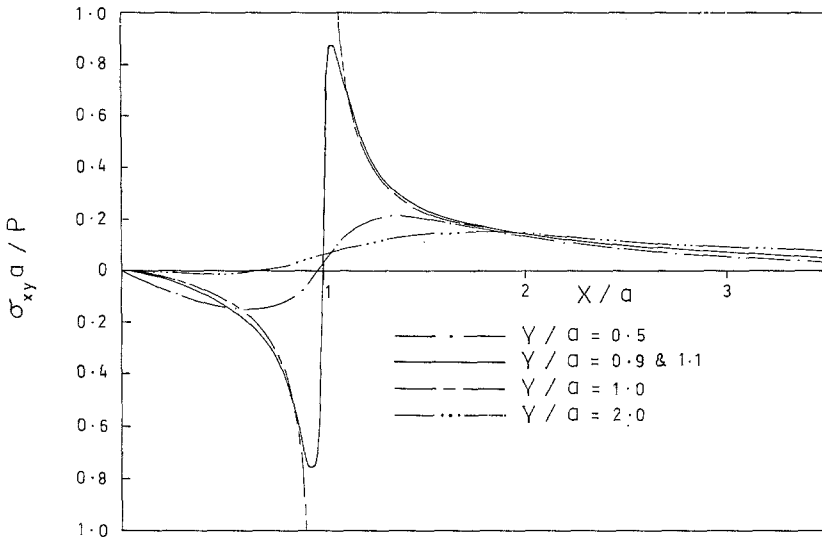


Fig. 12. Intensity factor. Variation of $\sigma_{xy} a / P$ with X/a for various Y/a ratios.

composite.

