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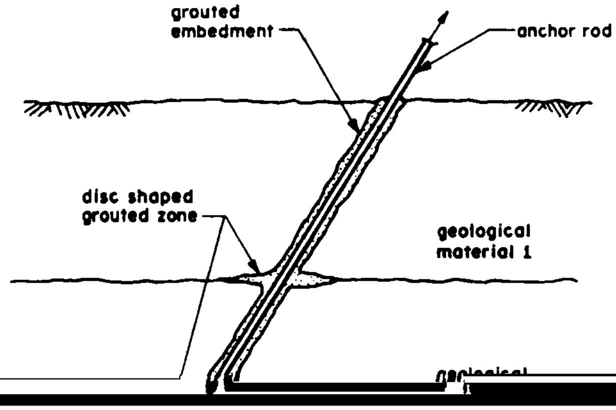
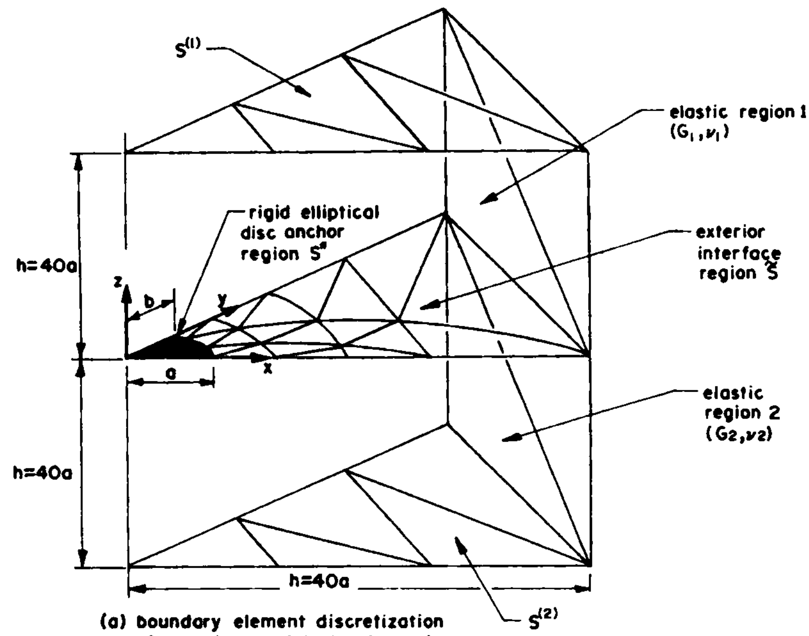
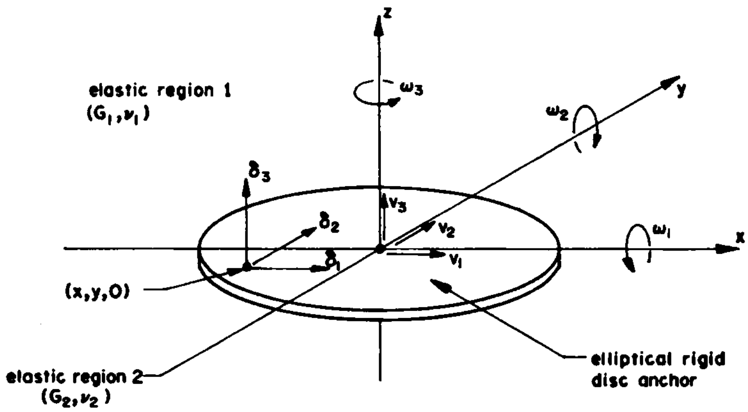

grouted embedment
anchor rod
disc shaped grouted zone
geological material 1



(a) boundary element discretization


z
elastic region 1
(G_1, ν_1)
ω_3
y
ω_2
δ_3
v_3
δ_2
v_2
ω_1
x
δ_1
v_1
(x,y,0)
elastic region 2
(G_2, ν_2)
elliptical rigid
disc anchor

[redacted]

can be obtained in the form

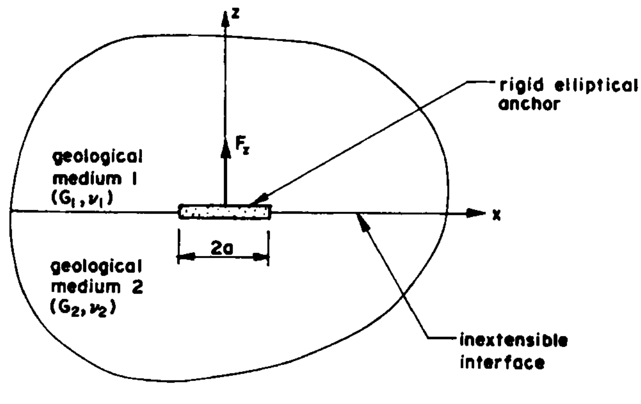
$$\{\mathbf{u}_{S^*}\} = [\mathbf{B}]\{\Delta\} \tag{14}$$

[redacted]

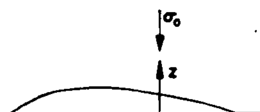
Thus (19) can be written in the form

$$
\{\mathbf{F}\} = -\begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & c_{34} & 0 & 0 \\ 0 & 0 & c_{43} & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} v_1 \\ \omega_2 \\ v_2 \\ \omega_1 \\ v_3 \\ \omega_3 \end{bmatrix} \tag{22}
$$

This formally completes the boundary element analysis of the problem of a rigid elliptical disc



(a) upper bound analysis



[redacted]

where C is an arbitrary constant and $e_0^2 = (a^2 - b^2)/a^2$. The variable u is related to the ellipsoidal co-ordinate ξ by

$$\xi^2 = a^2(sn^{-2}u - 1) \tag{53}$$

$$E(u) = \int_0^u \mathrm{d}n^2\, t\, \mathrm{d}t \tag{54}$$

The quantities $sn\,u$, $\mathrm{d}n\,u$, etc., represent the Jacobian elliptic functions[13] which have real and imaginary roots $4K$ and $2iK$, respectively, corresponding to the moduli e_0 and $e_0^1 = b/a$. It may also be noted that $E(e_0)$ is the complete elliptic integral of the second kind.[12] Considering the [redacted] stiffness of the elliptical rigid anchor embedded at the bi-material interface is identical to the equivalent set developed for the axial stiffness of the elliptical anchor (equation 41).

approximate solution (67) by virtue of the imposed constraint on u (given by (62)). Again, the

[redacted]

variables:

$$\bar{F}_z = \frac{F_z}{4\pi a\,\Delta_z(G_1 + G_2)/K(e_0)} \tag{70a}$$

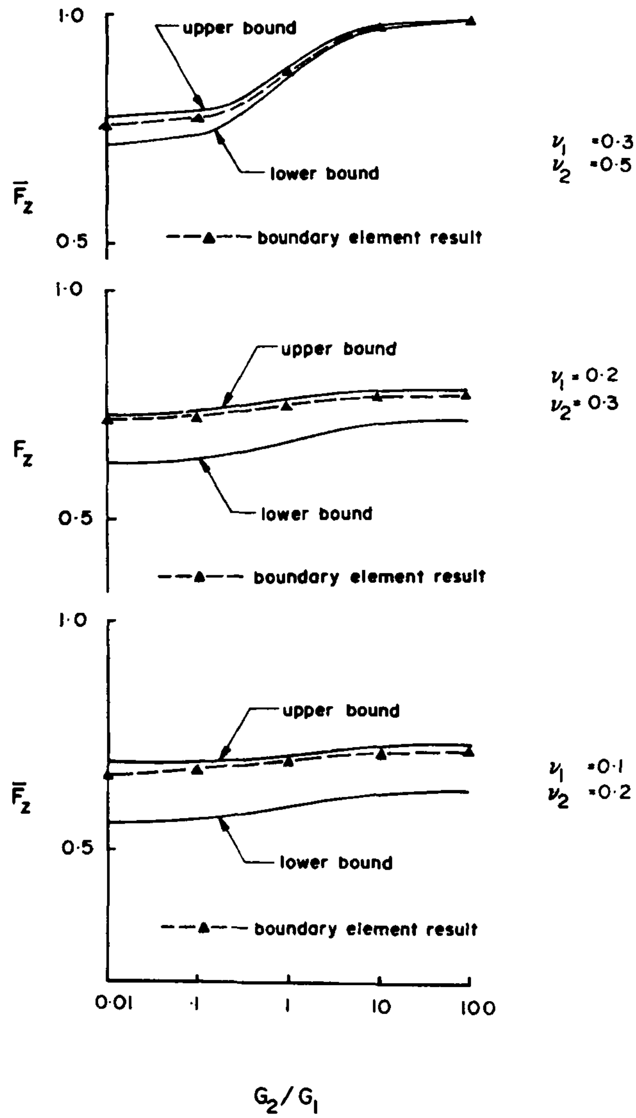
$$\bar{F}_x = \frac{F_x}{4\pi a\,\Delta_x(G_1 + G_2)e_0^2/3[K(e_0) - E(e_0)]} \tag{70b}$$

$$\bar{F}_x^* = \frac{F_x}{4\pi a^2\Omega_y^*(G_1 + G_2)e_0^2/3[K(e_0) - E(e_0)]} \tag{70c}$$

$$\bar{M}_y = \frac{M_y}{4\pi a^3\Omega_y(G_1 + G_2)e_0^2/3[K(e_0) - E(e_0)]} \tag{70d}$$

$$\bar{M}_z = \frac{M_z}{4\pi a^3\Omega_z(G_1 + G_2)/K(e_0)} \tag{70e}$$

[redacted]


1·0
upper bound
lower bound
$\bar{F}_z$
0·5
boundary element result
ν_1 = 0·3
ν_2 = 0·5
1·0
upper bound
ν_1 = 0·2
ν_2 = 0·3
F_z
lower bound
0·5
boundary element result
1·0
upper bound
ν_1 = 0·1
ν_2 = 0·2
$\bar{F}_z$
0·5
lower bound
boundary element result
0·01
·1
1
10
100
G_2/G_1

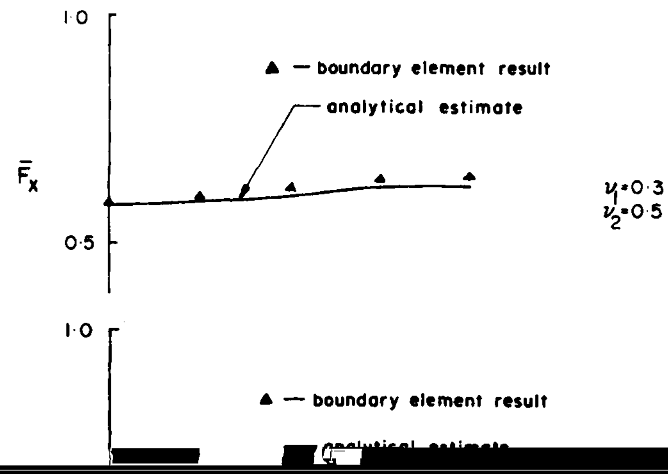
[redacted]

the bounds converge to a single result which agrees quite accurately with the boundary element estimate. Similar conclusions apply, in general, for the results for the non-dimensional rotational stiffness $\bar{M}_y$. Since the bounds for the axial stiffness are identical to the bounds for the rotational stiffness, the following relationship may be used to derive $M_y/\Omega_y a^2$ from F_z/Δ_z:

$$\frac{M_y}{\Omega_y a^2}=\frac{F_z K(e_0)e_0^2}{3\Delta_z\{K(e_0)-E(e_0)\}} \tag{71}$$

Relationship (71) applies only for the bounding estimates.

Figure 11 illustrates the boundary element results for the non-dimensional translational stiffness



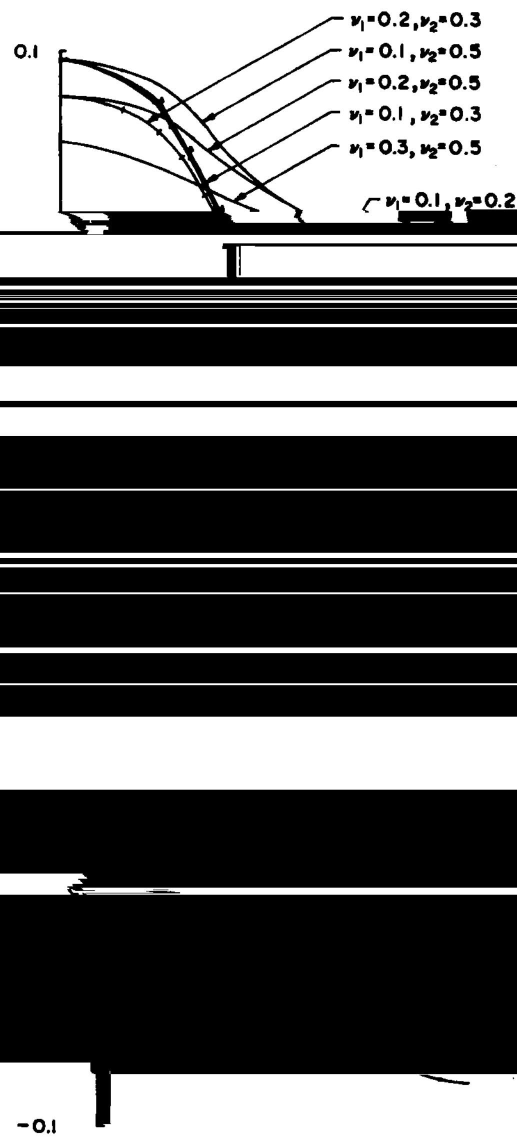


Figure 12 Coupling stiffness (rotation due to lateral force of a rigid elliptical anchor embedded at a bi-material

ACKNOWLEDGEMENTS

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